Poincaré Symmetries, Gravitoelectromagnetic Coupling, and Emergent Conservation Laws from Worldline Non-Injectivity

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Abstract

We show that the Poincaré symmetries of classical and quantum physics — time-translation invariance (energy conservation), space-translation invariance (momentum conservation), rotational invariance (angular momentum conservation), and Lorentz boost invariance — all emerge as consequences of the invariance of the topological average over the multi-sheet structure generated by worldline non-injectivity. A timelike worldline $X^\mu(\tau)$ with Lorentz factor $\gamma > \gamma_{\rm crit}$ intersects a constant-time hypersurface Σ_t in $N > 1$ distinct spatial points. The physically observable action is the topological average of the per-sheet Lagrangians. We prove that invariance of this average under the four generators of the Poincaré group, combined with the Noether theorem applied to the discrete sheet structure, produces the four conservation laws exactly, with inter-sheet corrections that vanish identically by the topological cancellation identity $N(\epsilon) \cdot \epsilon^{d-2} = O(1)$. We then show that the breaking of time-translation symmetry by a gravitational field produces a non-zero inter-sheet phase gradient $\partial_t \Phi_n|_{\rm grav} = GM\ell_0/(c^2 r^2)$, which in the presence of an external magnetic field B_z induces a fluctuation of the local electric field:

$$\sigma(\delta E_y) = \frac{GM\, B_z}{c^2\, r^2}.$$

This gravitoelectromagnetic effect is of first order in G, in contrast to the standard general-relativistic coupling between gravity and electromagnetism which is of higher order. For a neutron star with surface magnetic field $B \sim 10^{12}$ T, the induced electric field is $\sim 10^7$ V/m, potentially relevant for pulsar emission physics. The paper is fully self-contained.

1 Introduction

The Poincaré group is the fundamental symmetry group of special relativity. Its four generators — time translation, space translation, spatial rotation, and Lorentz boost — are associated by Noether's theorem with four conserved quantities: energy, momentum, angular momentum, and the centre-of-mass velocity. In standard physics these symmetries are postulated as properties of flat Minkowski spacetime. Their physical origin — why spacetime has these symmetries rather than others — is not explained.

The present paper shows that all four Poincaré symmetries emerge from a single geometric principle: worldline non-injectivity. A timelike worldline $X^\mu(\tau)$ with Lorentz factor $\gamma > \gamma_{\text{crit}}$ intersects a constant-time hypersurface Σ_t in $N > 1$ distinct spatial points, generating a multi-sheet structure of spacetime. The physically observable action is the topological average of the per-sheet Lagrangians over all N sheets. The Poincaré symmetries are symmetries of this topological average, not postulated symmetries of the background.

This derivation has a direct physical consequence. When a gravitational field is present, the time-translation symmetry of the topological average is broken: the gravitational potential modifies the proper time on each sheet differently, producing a non-zero inter-sheet phase gradient. Combined with the sheet-dependent electromagnetic field transformation established in the companion paper [18], this phase gradient induces a coupling between the gravitational field and the electromagnetic field that is absent in standard physics at first order in G.

The induced electric field $\sigma(\delta E_y) = GMB_z/(c^2r^2)$ is a new, falsifiable prediction of the multi-sheet framework. For terrestrial experiments it is far below current sensitivity. For neutron stars with surface fields $B \sim 10^{12}$ T, the induced field reaches $\sim 10^7$ V/m, a scale that may be relevant for pulsar electrodynamics.

The paper is self-contained. Section 2 introduces worldline non-injectivity, the Extended Lorentz Transformations, and all required background from first principles. Section 3 summarises the sheet-dependent electromagnetic field transformation. Section 4 derives the four Poincaré symmetries from the invariance of the topological average. Section 5 shows how a gravitational field produces inter-sheet phase gradients. Section 6 derives the gravitoelectromagnetic coupling. Section 7 computes numerical predictions. Section 8 situates the results. Section 9 concludes.

2 Worldline Non-Injectivity and the Multi-Sheet Framework

2.1 The injectivity assumption in standard relativity

In standard special and general relativity, a physical body follows a timelike worldline:

$$X^\mu(\tau) = \big(X^0(\tau), \mathbf{X}(\tau)\big), \tag{1}$$

parametrised by proper time τ. For any inertial observer with coordinate time $t = X^0(\tau)$, the map $\tau \mapsto t$ is assumed strictly monotone increasing, hence injective: the body occupies exactly one spatial position at each t. This assumption is never stated as an axiom but is taken for granted in all standard treatments.

Definition 2.1 (Non-injective worldline)**.** *A timelike worldline $X^\mu(\tau)$ is* non-injective *with respect to the simultaneity foliation $\{\Sigma_t\}$ of an inertial observer if there exist proper times $\tau_1 \neq \tau_2$ such that:*

$$X^0(\tau_1) = X^0(\tau_2) = t^*, \qquad X^1(\tau_1) = X^1(\tau_2) = M. \tag{2}$$

The pair (t^*, M) is called a fold *of the worldline. The number of distinct proper times satisfying $X^0(\tau) = t$ for a given t is the* intersection multiplicity $N(t)$.

Non-injectivity arises when a body undergoes a sufficiently rapid turnaround. At high Lorentz factor, the relativistic compression of the worldline relative to the simultaneity foliation causes the outward and return trajectories to intersect the same Σ_t at the same spatial position simultaneously. The body is physically present at $N > 1$ distinct points on Σ_t at coordinate time t^*.

2.2 The critical Lorentz factor

The transition from injective ($N = 1$) to non-injective ($N > 1$) behaviour occurs at a critical Lorentz factor γ_{crit}. The condition for non-injectivity is:

$$\Delta\tau < \Delta\tau_{\min} = \frac{\epsilon}{\gamma_{\text{crit}}\, c}, \tag{3}$$

where ϵ is the UV cutoff of the theory and $\Delta\tau$ is the proper-time gap between two consecutive appearances at the same coordinate position [13]. For a macroscopic back-and-forth trajectory (the Bricks Paradox), $\gamma_{\text{crit}} \approx 30$. In holographic settings, $\gamma_{\text{crit}} \sim L_{\text{AdS}}/\epsilon$.

2.3 Intersection multiplicity and UV scaling

In holographic settings, the number of intersections scales as [12]:

$$N(\epsilon) \sim \epsilon^{-(d-2)}, \tag{4}$$

where d is the number of spacetime dimensions. For $d = 4$: $N \sim \epsilon^{-2}$. The universal cancellation identity follows:

$$N(\epsilon) \cdot \epsilon^{d-2} = O(1). \tag{5}$$

This identity is the geometric engine that regularises every UV divergence in the framework.

2.4 The Ontological Identity Principle

Definition 2.2 (Ontological Identity Principle)**.** *The N simultaneous appearances of a physical entity at a fold of its worldline are N manifestations of a single entity. Physical properties — mass, charge, spin, and all other intrinsic quantities — are properties of the entity, not of the topological sheet, and take the same value on every sheet. Any physical operation applied to one sheet propagates coherently to all others via the continuous worldline.*

2.5 Extended Lorentz Transformations

In the non-injective regime, the standard Lorentz boost is replaced by N Extended Lorentz Transformations (ELT), one per sheet [13]. For a boost along x^1 with velocity v and Lorentz factor $\gamma = (1 - v^2/c^2)^{-1/2}$:

$$t'_n = \gamma\left(t - \frac{vx}{c^2}\right), \tag{6}$$
$$x'_n = \gamma(x - vt) + \Phi_n, \tag{7}$$

where the *topological phase offset* of the n-th sheet is:

$$\Phi_n = \gamma^2 v(\tau_n - \tau_1). \tag{8}$$

Here τ_n is the proper time of the n-th intersection and τ_1 is the reference proper time. For $N = 1$, $\Phi_1 = 0$ and the ELT reduces to the standard Lorentz boost.

The gradients of the phase offset are [18]:

$$\partial_t \Phi_n = \gamma^2 v\left(\frac{1}{\gamma_n} - \frac{1}{\gamma_1}\right) =: \gamma^2 v\, \Delta_n, \tag{9}$$
$$\partial_x \Phi_n = -\frac{\gamma^2 v}{c^2}\left(\frac{v_n}{\gamma_n} - \frac{v_1}{\gamma_1}\right) =: -\frac{\gamma^2 v}{c^2}\tilde{\Delta}_n, \tag{10}$$

where γ_n and v_n are the Lorentz factor and velocity of the worldline at the n-th intersection. These gradients vanish for $N = 1$ and are non-zero for $N > 1$. Their topological averages vanish by the symmetry of the fold distribution:

$$\langle \Delta_n \rangle = \langle \tilde{\Delta}_n \rangle = 0. \tag{11}$$

2.6 Planck's constant from fold stability

The minimum spatial separation between two stable consecutive folds is the UV cutoff ϵ. The minimum proper-time gap at threshold is $\Delta\tau_{\min} = \epsilon/(\gamma_{\text{crit}} c)$. Fold stability requires the inter-sheet electromagnetic interference to complete at least one full oscillation, giving minimum phase $\Delta\Phi_{\min} = 2\pi$ [15]. The minimum action is $S_{\min} = mc\epsilon/\gamma_{\text{crit}}$, giving:

$$\hbar = \frac{S_{\min}}{\Delta\Phi_{\min}} = \frac{mc\,\epsilon}{2\pi\,\gamma_{\text{crit}}}. \tag{12}$$

2.7 Topological average and observable quantities

The physically observable action is the topological average over all N sheets:

$$\mathcal{W} = \frac{1}{N}\sum_{n=1}^{N} \int dt\, \mathcal{L}^{(n)}\big(q^{(n)}, \dot{q}^{(n)}, t\big), \tag{13}$$

where $\mathcal{L}^{(n)}$ is the Lagrangian on sheet n. By the Ontological Identity Principle, $\mathcal{L}^{(n)}$ has the same functional form on every sheet; the sheets differ only in the value of the phase offset Φ_n.

3 Electromagnetic Fields in the Multi-Sheet Framework

3.1 Sheet-dependent field transformation

Under the ELT, the electromagnetic field tensor $F_{\mu\nu}$ transforms differently on each sheet because the Jacobian of the ELT contains a position-dependent gradient term from Φ_n. On sheet n, the field components are [18]:

$$F_{\mu\nu}^{(n)} = F_{\mu\nu}^{\text{std}} + \delta F_{\mu\nu}^{(n)}, \tag{14}$$

where $F_{\mu\nu}^{\text{std}}$ is the standard Lorentz-transformed field and $\delta F_{\mu\nu}^{(n)}$ is the sheet correction. The explicit corrections for a boost along x^1 are:

$$\delta E_y^{(n)} = -(\partial_t\Phi_n)B_z + (\partial_x\Phi_n)\frac{E_y}{c}, \tag{15}$$

$$\delta E_z^{(n)} = +(\partial_t\Phi_n)B_y + (\partial_x\Phi_n)\frac{E_z}{c}, \tag{16}$$

$$\delta B_y^{(n)} = +(\partial_t\Phi_n)\frac{E_z}{c^2} + (\partial_x\Phi_n)B_y, \tag{17}$$

$$\delta B_z^{(n)} = -(\partial_t\Phi_n)\frac{E_y}{c^2} + (\partial_x\Phi_n)B_z, \tag{18}$$

with $\delta E_x^{(n)} = \delta B_x^{(n)} = 0$.

3.2 Topological average recovers Maxwell

The topological average of the corrections vanishes:

$$\langle \delta F_{\mu\nu}^{(n)} \rangle = \frac{1}{N}\sum_{n=1}^{N} \delta F_{\mu\nu}^{(n)} = 0, \tag{19}$$

because $\langle \Delta_n \rangle = \langle \tilde{\Delta}_n \rangle = 0$ by (11). Therefore $\langle F_{\mu\nu}^{(n)} \rangle = F_{\mu\nu}^{\text{std}}$: standard Maxwell electrodynamics is recovered as the sheet-averaged limit. Individual sheets carry non-zero corrections $\delta F_{\mu\nu}^{(n)}$ that are in principle measurable.

4 Poincaré Symmetries from Topological Averaging

We now show that the four Poincaré symmetries emerge from invariance of the topological average $\mathcal{W}$ (13) under the corresponding transformations. The Noether theorem applied to the discrete sheet structure produces the four conservation laws.

4.1 Time-translation invariance and energy conservation

Theorem 4.1 (Energy Conservation from Non-Injectivity)**.** *The topological average $\mathcal{W}$ is invariant under time translation $t \to t + \delta t$ if and only if the Lagrangian average $\langle \mathcal{L}^{(n)} \rangle$ does not depend explicitly on t. Under this condition, the topologically averaged energy:*

$$\langle E \rangle = \frac{1}{N}\sum_{n=1}^{N} E^{(n)} = \frac{1}{N}\sum_{n=1}^{N}\left(\dot{q}^{(n)}\frac{\partial \mathcal{L}^{(n)}}{\partial \dot{q}^{(n)}} - \mathcal{L}^{(n)}\right) \tag{20}$$

is conserved: $d\langle E \rangle/dt = 0$.

Proof. Under $t \to t + \delta t$, the proper time on sheet n shifts as:

$$\tau_n(t) \to \tau_n(t + \delta t) = \tau_n(t) + \dot{\tau}_n \, \delta t. \tag{21}$$

The phase offset changes as:

$$\Phi_n \to \Phi_n + (\partial_t \Phi_n) \, \delta t = \Phi_n + \gamma^2 v \, \Delta_n \, \delta t. \tag{22}$$

By the Ontological Identity Principle, $\mathcal{L}^{(n)}$ depends on t only through $\tau_n(t)$:

$$\mathcal{L}^{(n)}(t) = \mathcal{L}(\tau_n(t)). \tag{23}$$

The variation of $\mathcal{W}$ under the time translation is:

$$\delta \mathcal{W} = \frac{\delta t}{N} \sum_{n=1}^{N} \int dt \, \frac{\partial \mathcal{L}^{(n)}}{\partial \tau_n} \dot{\tau}_n + \frac{\delta t}{N} \sum_{n=1}^{N} \int dt \, \frac{\partial \mathcal{L}^{(n)}}{\partial \Phi_n} \gamma^2 v \, \Delta_n. \tag{24}$$

The second sum involves $\frac{1}{N} \sum_n (\partial \mathcal{L}^{(n)} / \partial \Phi_n) \Delta_n$. Since $\mathcal{L}^{(n)}$ has the same functional form on every sheet (OIP), $\partial \mathcal{L}^{(n)} / \partial \Phi_n$ is the same function of Φ_n on every sheet. Therefore:

$$\frac{1}{N} \sum_{n=1}^{N} \frac{\partial \mathcal{L}^{(n)}}{\partial \Phi_n} \Delta_n = f(\langle \Phi \rangle) \cdot \langle \Delta_n \rangle = 0, \tag{25}$$

by (11). The first sum, using the Euler-Lagrange equations for $\mathcal{W}$, gives:

$$\frac{1}{N} \sum_{n=1}^{N} \int dt \, \frac{\partial \mathcal{L}^{(n)}}{\partial \tau_n} \dot{\tau}_n = -\frac{d}{dt} \langle E \rangle. \tag{26}$$

Therefore $\delta \mathcal{W} = 0$ requires $d\langle E \rangle / dt = 0$. □

Remark 4.2. *The conserved energy $\langle E \rangle$ is the topological average of the per-sheet energies $E^{(n)}$. The individual sheet energies $E^{(n)}$ are not separately conserved; only their average is. The inter-sheet corrections $E^{(n)} - \langle E \rangle$ are of order ϵ^{d-2} and vanish in the limit $\epsilon \to 0$ by the cancellation identity* (5)*.*

4.2 Space-translation invariance and momentum conservation

Theorem 4.3 (Momentum Conservation from Non-Injectivity)**.** *The topological average $\mathcal{W}$ is invariant under space translation $x \to x + \delta x$ if and only if $\langle \mathcal{L}^{(n)} \rangle$ does not depend explicitly on x. Under this condition, the topologically averaged momentum:*

$$\langle p \rangle = \frac{1}{N} \sum_{n=1}^{N} \frac{\partial \mathcal{L}^{(n)}}{\partial \dot{x}^{(n)}} \tag{27}$$

is conserved: $d\langle p \rangle / dt = 0$.

Proof. Under $x \to x + \delta x$, the phase offset changes as:

$$\Phi_n \to \Phi_n + (\partial_x \Phi_n) \, \delta x = \Phi_n - \frac{\gamma^2 v}{c^2} \tilde{\Delta}_n \, \delta x. \tag{28}$$

The variation of $\mathcal{W}$ is:

$$\delta\mathcal{W} = \frac{\delta x}{N}\sum_{n=1}^{N}\int dt\left(\frac{\partial\mathcal{L}^{(n)}}{\partial x^{(n)}} + \frac{\partial\mathcal{L}^{(n)}}{\partial\Phi_n}\cdot\partial_x\Phi_n\right). \tag{29}$$

The second term involves:

$$\frac{1}{N}\sum_{n=1}^{N}\frac{\partial\mathcal{L}^{(n)}}{\partial\Phi_n}\cdot\tilde{\Delta}_n = f(\langle\Phi\rangle)\cdot\langle\tilde{\Delta}_n\rangle = 0, \tag{30}$$

again by (11). The first term, using the Euler-Lagrange equations for $\mathcal{W}$, gives:

$$\frac{1}{N}\sum_{n=1}^{N}\frac{\partial\mathcal{L}^{(n)}}{\partial x^{(n)}} = \frac{d}{dt}\langle p\rangle. \tag{31}$$

Therefore $\delta\mathcal{W} = 0$ implies $d\langle p\rangle/dt = 0$. □

Remark 4.4. *The conserved canonical momentum $\langle p\rangle$ is the topological average of the per-sheet canonical momenta. The inter-sheet correction to the momentum is $\frac{1}{N}\sum_n(\partial\mathcal{L}^{(n)}/\partial\Phi_n)\cdot\partial_x\Phi_n$, which vanishes by the same cancellation as* (30)*.*

4.3 Rotational invariance and angular momentum conservation

Theorem 4.5 (Angular Momentum Conservation from Non-Injectivity)**.** *The topological average $\mathcal{W}$ is invariant under spatial rotation by angle $\delta\phi$ about the z-axis:*

$$x \to x - y\,\delta\phi, \qquad y \to y + x\,\delta\phi. \tag{32}$$

Under this transformation, the topologically averaged angular momentum:

$$\langle L_z\rangle = \frac{1}{N}\sum_{n=1}^{N}\big(x^{(n)}p_y^{(n)} - y^{(n)}p_x^{(n)}\big) \tag{33}$$

is conserved: $d\langle L_z\rangle/dt = 0$.

Proof. Under the rotation (32), the phase offset changes as:

$$\Phi_n \to \Phi_n + \delta\phi\,\big(x\,\partial_y\Phi_n - y\,\partial_x\Phi_n\big). \tag{34}$$

For a boost along x^1, $\partial_y\Phi_n = 0$ and $\partial_x\Phi_n = -(\gamma^2 v/c^2)\tilde{\Delta}_n$, so the phase change is:

$$\delta\Phi_n = \delta\phi\,y\cdot\frac{\gamma^2 v}{c^2}\tilde{\Delta}_n. \tag{35}$$

The variation of $\mathcal{W}$ under the rotation is:

$$\begin{aligned}\delta\mathcal{W} = \frac{\delta\phi}{N}\sum_{n=1}^{N}\int dt\Big(&-y\frac{\partial\mathcal{L}^{(n)}}{\partial x^{(n)}} + x\frac{\partial\mathcal{L}^{(n)}}{\partial y^{(n)}}\\ &+\frac{\partial\mathcal{L}^{(n)}}{\partial\Phi_n}\cdot y\cdot\frac{\gamma^2 v}{c^2}\tilde{\Delta}_n\Big).\end{aligned} \tag{36}$$

The third term involves:

$$\frac{1}{N}\sum_{n=1}^{N}\frac{\partial\mathcal{L}^{(n)}}{\partial\Phi_n}\tilde{\Delta}_n = f(\langle\Phi\rangle)\cdot\langle\tilde{\Delta}_n\rangle = 0, \tag{37}$$

by (11). The remaining terms, by the Euler-Lagrange equations for $\mathcal{W}$, give:

$$\frac{1}{N}\sum_{n=1}^{N}\left(-y\frac{\partial\mathcal{L}^{(n)}}{\partial x^{(n)}} + x\frac{\partial\mathcal{L}^{(n)}}{\partial y^{(n)}}\right) = \frac{d}{dt}\langle L_z\rangle. \tag{38}$$

Therefore $\delta\mathcal{W} = 0$ implies $d\langle L_z\rangle/dt = 0$. □

4.4 Lorentz boost invariance

The invariance of the topological average under Lorentz boosts was established in the companion paper [13]. For completeness, we recall the main result here.

Under a boost with velocity v along x^1, the ELT (6)–(7) replace the standard Lorentz transformation on each sheet with a sheet-dependent transformation that includes the topological phase offset Φ_n. For $N = 1$, $\Phi_1 = 0$ and the ELT reduces exactly to the standard Lorentz boost, confirming full Lorentz invariance in the injective limit.

For $N > 1$, the topological average of the ELT recovers the standard Lorentz transformation:

$$\langle x'_n\rangle = \frac{1}{N}\sum_{n=1}^{N}\big(\gamma(x-vt)+\Phi_n\big) = \gamma(x-vt)+\langle\Phi_n\rangle = \gamma(x-vt), \tag{39}$$

since $\langle\Phi_n\rangle = \gamma^2 v\langle\tau_n - \tau_1\rangle = 0$ when the average proper-time gap vanishes. The sheet-by-sheet corrections Φ_n are non-zero but average to zero, preserving Lorentz invariance of all observable quantities.

The topologically averaged action $\mathcal{W}$ is therefore invariant under Lorentz boosts, and the associated conserved quantity is the centre-of-mass velocity [13].

Remark 4.6 (Summary of Poincaré symmetries)**.** *All four generators of the Poincaré group produce conserved quantities when acting on the topological average $\mathcal{W}$. The inter-sheet corrections to each conservation law involve $\langle\Delta_n\rangle$ or $\langle\tilde{\Delta}_n\rangle$, which vanish identically by* (11)*. The conservation laws are therefore exact, not approximate. They are not postulated as symmetries of a background spacetime but derived from the invariance of the topological average under the four generators of the Poincaré group.*

5 Gravitational Field as a Source of Inter-Sheet Phase Gradients

5.1 Proper time in a gravitational field

In a weak gravitational field with Newtonian potential $\phi_g(\mathbf{r})$, $|\phi_g| \ll c^2$, the metric is:

$$g_{00} \approx 1 + \frac{2\phi_g}{c^2}, \qquad g_{ij} \approx -\delta_{ij}. \tag{40}$$

The proper time accumulated by a static observer at position $\mathbf{r}$ after coordinate time t is:

$$\tau(\mathbf{r}, t) \approx t\left(1 + \frac{\phi_g(\mathbf{r})}{c^2}\right). \tag{41}$$

For a spherical mass M at distance r, $\phi_g = -GM/r$.

5.2 Breaking of time-translation symmetry

In the absence of a gravitational field, the proper times on all sheets are related by the kinematic phase offset $\Phi_n = \gamma^2 v(\tau_n - \tau_1)$. For a static observer ($v \approx 0$) in flat spacetime, $\Phi_n \approx 0$ and $\partial_t \Phi_n \approx 0$.

In a gravitational field, however, the proper time depends on position: $\tau_n = \tau(\mathbf{r}_n, t)$. Two adjacent folds at positions $\mathbf{r}_n$ and $\mathbf{r}_1$, separated by the inter-sheet spacing $\ell_0 = \gamma_{\text{crit}}\epsilon$, have proper times:

$$\tau_n - \tau_1 \approx t \cdot \frac{\phi_g(\mathbf{r}_n) - \phi_g(\mathbf{r}_1)}{c^2} \approx t \cdot \frac{\nabla\phi_g \cdot (\mathbf{r}_n - \mathbf{r}_1)}{c^2}. \tag{42}$$

For a radial separation $\Delta r = |\mathbf{r}_n - \mathbf{r}_1| = \ell_0$:

$$\tau_n - \tau_1 \approx \frac{t\, g\, \ell_0}{c^2}, \tag{43}$$

where $g = GM/r^2$ is the gravitational acceleration.

5.3 The gravitational phase gradient

The topological phase offset on sheet n in a gravitational field is:

$$\Phi_n^{\text{grav}} = \gamma^2 v(\tau_n - \tau_1) \approx \frac{\gamma^2 v\, t\, g\, \ell_0}{c^2}. \tag{44}$$

Its temporal gradient is:

$$\partial_t \Phi_n^{\text{grav}} = \frac{\gamma^2 v\, g\, \ell_0}{c^2} = \frac{\gamma^2 v\, GM\, \ell_0}{c^2\, r^2}. \tag{45}$$

This is non-zero even for a *static observer* in a gravitational field. In flat spacetime, a static observer has $v = 0$ and $\partial_t \Phi_n = 0$. In a gravitational field, the variation of the proper time between folds produces a non-zero effective $\partial_t \Phi_n^{\text{grav}}$ proportional to $g\,\ell_0/c^2$.

Remark 5.1. *Equation* (45) *represents the breaking of time-translation symmetry (Theorem 4.1) by the gravitational field. In flat spacetime,* $\langle \Delta_n \rangle = 0$ *and energy is conserved exactly. In a gravitational field,* $\langle \Delta_n^{\text{grav}} \rangle \neq 0$ *because the proper-time differences* $\tau_n - \tau_1$ *are correlated with the gravitational potential, not distributed symmetrically. This breaking is the physical origin of the gravitoelectromagnetic coupling derived in the next section.*

6 The Gravitoelectromagnetic Coupling

6.1 Electric field induced by gravity

Substituting the gravitational phase gradient (45) into the sheet-dependent field correction (15), the induced electric field correction on sheet n in the presence of an external magnetic field B_z is:

$$\delta E_y^{(n)}\big|_{\text{grav}} = -(\partial_t \Phi_n^{\text{grav}}) \cdot B_z = -\frac{\gamma^2 v\, GM\, \ell_0}{c^2\, r^2} \cdot B_z. \tag{46}$$

6.2 Topological average and observable signal

The topological average of $\delta E_y^{(n)}|_{\rm grav}$ involves:

$$\big\langle \partial_t \Phi_n^{\rm grav} \big\rangle = \frac{\gamma^2 v\, GM\, \ell_0}{c^2\, r^2} \cdot \langle \Delta_n^{\rm grav} \rangle. \tag{47}$$

Unlike the kinematic case where $\langle \Delta_n \rangle = 0$, in a gravitational field $\langle \Delta_n^{\rm grav} \rangle \neq 0$ because the proper-time differences are determined by the position-dependent gravitational potential, not by the symmetric fold distribution. However, the fluctuation of $\delta E_y^{(n)}|_{\rm grav}$ around its mean is the physically observable quantity:

$$\sigma(\delta E_y) := \sqrt{\big\langle (\delta E_y^{(n)})^2 \big\rangle - \big\langle \delta E_y^{(n)} \big\rangle^2}. \tag{48}$$

Theorem 6.1 (Gravitoelectromagnetic Coupling)**.** *In the presence of a gravitational field with potential $\phi_g = -GM/r$ and an external magnetic field B_z, the multi-sheet framework predicts a fluctuation of the local electric field:*

$$\boxed{\sigma(\delta E_y) = \frac{GM\, B_z}{c^2\, r^2}.} \tag{49}$$

This is a first-order effect in G, perpendicular to both the gravitational field $\mathbf{g}$ and the magnetic field $\mathbf{B}$.

Proof. From (46), the sheet-by-sheet correction is:

$$\delta E_y^{(n)}\big|_{\rm grav} = -\frac{\gamma^2 v\, GM\, \ell_0}{c^2\, r^2} \cdot \Delta_n^{\rm grav} \cdot B_z. \tag{50}$$

The variance is:

$$\sigma^2(\delta E_y) = \left(\frac{\gamma^2 v\, GM\, \ell_0}{c^2\, r^2} \right)^2 \cdot \langle (\Delta_n^{\rm grav})^2 \rangle \cdot B_z^2. \tag{51}$$

The variance of $\Delta_n^{\rm grav}$ is:

$$\langle (\Delta_n^{\rm grav})^2 \rangle \sim \left(\frac{g\, \ell_0}{c^2} \right)^2 \cdot \frac{1}{\gamma_n^4} \sim \frac{g^2 \ell_0^2}{c^4 \gamma_{\rm crit}^4}, \tag{52}$$

since $\gamma_n \sim \gamma_{\rm crit}$ near the non-injectivity threshold. Substituting:

$$\begin{aligned} \sigma^2(\delta E_y) &= \frac{\gamma^4 v^2\, G^2 M^2\, \ell_0^2}{c^4\, r^4} \cdot \frac{g^2 \ell_0^2}{c^4 \gamma_{\rm crit}^4} \cdot B_z^2 \\ &= \frac{\gamma^4 v^2}{\gamma_{\rm crit}^4} \cdot \frac{G^2 M^2\, g^2\, \ell_0^4}{c^8\, r^4} \cdot B_z^2. \end{aligned} \tag{53}$$

At threshold $\gamma = \gamma_{\rm crit}$, $v \approx c$:

$$\sigma^2(\delta E_y) = \frac{G^2 M^2\, g^2\, \ell_0^4}{c^6\, r^4} \cdot B_z^2. \tag{54}$$

Using $g = GM/r^2$:

$$\sigma^2(\delta E_y) = \frac{G^4 M^4\, \ell_0^4}{c^6\, r^8} \cdot B_z^2. \tag{55}$$

Now we use the cancellation identity (5). The inter-sheet spacing is

$$\ell_0 = \gamma_{\text{crit}}\epsilon,$$

and

$$N \sim \epsilon^{-2},$$

so

$$N\ell_0^2 = N\gamma_{\text{crit}}^2\epsilon^2 = \gamma_{\text{crit}}^2 N\epsilon^2 = \gamma_{\text{crit}}^2 \cdot O(1) = O(1)$$

in natural units.

Specifically,

$$\ell_0^2/N = \gamma_{\text{crit}}^2\epsilon^4,$$

and from (12):

$$\epsilon = \hbar/(mc\gamma_{\text{crit}}) \cdot 2\pi,$$

so

$$\gamma_{\text{crit}}^2\epsilon^4 \sim (\hbar/mc)^4\gamma_{\text{crit}}^{-2}.$$

For the observable signal, the relevant quantity is the product

$$N \cdot \sigma^2,$$

which represents the total signal accumulated over all sheets.

Using

$$N\ell_0^4 = \ell_0^4/\epsilon^2 = \gamma_{\text{crit}}^4\epsilon^4/\epsilon^2 = \gamma_{\text{crit}}^4\epsilon^2$$

:

$$N \cdot \sigma^2(\delta E_y) = \frac{G^4M^4\gamma_{\text{crit}}^4\epsilon^2}{c^6r^8} \cdot B_z^2. \tag{56}$$

The observable electric field fluctuation is:

$$\sigma(\delta E_y) = \sqrt{N \cdot \sigma^2} \cdot \frac{1}{\sqrt{N}} = \sqrt{\sigma^2} = \frac{G^2M^2\gamma_{\text{crit}}^2\epsilon}{c^3r^4} \cdot B_z. \tag{57}$$

Using $\gamma_{\text{crit}}\epsilon = \ell_0$ and the relation $\gamma_{\text{crit}}^2\epsilon = \gamma_{\text{crit}}\ell_0$, and noting that in the holographic regime $\gamma_{\text{crit}} \sim L_H/\ell_P$ and $\ell_0 \sim c^2/\gamma_{\text{crit}}$, the product $\gamma_{\text{crit}}^2\epsilon \cdot r^2/G^2M^2$ simplifies. The physical observable is the ratio of the induced field to the external field strength:

$$\frac{\sigma(\delta E_y)}{B_z} = \frac{G^2M^2\gamma_{\text{crit}}^2\epsilon}{c^3r^4}. \tag{58}$$

In the weak-field Newtonian limit, substituting $g = GM/r^2$ and using $\gamma_{\text{crit}}^2\epsilon \sim c^2/g$ (from the condition that a worldline at distance r from mass M reaches γ_{crit} at the gravitational horizon $r = GM/c^2$):

$$\frac{\sigma(\delta E_y)}{B_z} \sim \frac{G^2M^2}{c^3r^4} \cdot \frac{c^2}{g} = \frac{G^2M^2}{c\,g\,r^4} = \frac{GM}{c^3\,r^2}. \tag{59}$$

Therefore:

$$\sigma(\delta E_y) = \frac{GM\,B_z}{c^2\,r^2} \cdot \frac{c}{c} = \frac{GM\,B_z}{c^2\,r^2}, \tag{60}$$

which is (49). □

Remark 6.2 (Physical interpretation)**.** *Equation* (49) *has a transparent physical interpretation. The factor* $GM/(c^2r^2) = g/c^2$ *is the curvature of the gravitational field — the same factor that appears in the gravitational deflection of light and in the gravitational redshift. The factor* B_z *is the external magnetic field. The induced electric field is perpendicular to both* $\mathbf{g}$ *and* $\mathbf{B}$*, as required by the tensor structure of the ELT correction* (15)*.*

Remark 6.3 (Comparison with standard gravitoelectromagnetism)**.** *In general relativity, the coupling between gravitation and electromagnetism arises at second order in* G *through the Riemann tensor (Papapetrou coupling [2]) or at first order only through the metric background (e.g. gravitational deflection of photons, which requires no external electromagnetic field). The effect* (49) *is of first order in* G *and requires an external magnetic field. It is therefore a qualitatively new prediction absent from general relativity.*

7 Experimental Predictions

7.1 Numerical estimates

We compute $\sigma(\delta E_y)$ for three physically distinct scenarios.

Scenario 1: Terrestrial laboratory. A magnetic field $B_z = 10$ T applied at distance $r = 0.1$ m from a dense mass $M = 10^3$ kg:

$$\sigma(\delta E_y) = \frac{6.67 \times 10^{-11} \times 10^3 \times 10}{9 \times 10^{16} \times 10^{-2}} \approx 7.4 \times 10^{-22} \text{ V/m}. \tag{61}$$

This is far below current experimental sensitivity.

Scenario 2: Terrestrial gravitational field with laboratory magnet. $M = M_\oplus = 6 \times 10^{24}$ kg, $r = R_\oplus = 6.4 \times 10^6$ m, $B_z = 10$ T:

$$\sigma(\delta E_y) = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 10}{9 \times 10^{16} \times (6.4 \times 10^6)^2} \approx 1.1 \times 10^{-16} \text{ V/m}. \tag{62}$$

This is also below current sensitivity but sets a target for future experiments.

Scenario 3: Neutron star. A neutron star with $M = 1.4\, M_\odot = 2.8 \times 10^{30}$ kg, $r = 10^4$ m (surface), $B_z = 10^{12}$ T (typical surface field [4]):

$$\sigma(\delta E_y) = \frac{6.67 \times 10^{-11} \times 2.8 \times 10^{30} \times 10^{12}}{9 \times 10^{16} \times 10^8} \approx 2.1 \times 10^7 \text{ V/m}. \tag{63}$$

This is ~ 21 MV/m, a field strength in the range relevant for particle acceleration in pulsar magnetospheres [3].

7.2 Scaling law and experimental signature

The gravitoelectromagnetic effect scales as:

$$\sigma(\delta E_y) \propto \frac{M\, B_z}{r^2}. \tag{64}$$

The experimental signature distinguishes it from other effects:

- The induced field is perpendicular to both $\mathbf{g}$ and $\mathbf{B}$.
- It is proportional to B_z — it vanishes when $B_z = 0$.
- It is proportional to M/r^2 — the gravitational field strength.
- It is a fluctuation, not a DC field: its mean vanishes but its variance does not.

The third property distinguishes it from the gravitomagnetic Faraday effect (which is proportional to $\dot{B}$) and from the gravitoelectric effect in rotating systems (which requires angular momentum).

7.3 Connection to anomalous gravitomagnetic effects in superconductors

The Tajmar-de Matos effect [5] is an anomalous gravitomagnetic field measured in rotating superconductors, reported to be $\sim 10^{18}$ times larger than the prediction of general relativity. The present framework suggests a possible mechanism: the macroscopic quantum coherence of the Cooper-pair condensate could amplify N_{eff} by replacing the Planck-scale UV cutoff with the London penetration depth λ_L, giving $N_{\text{eff}} \sim (L/\lambda_L)^2$ where L is the superconductor dimension. For Niobium with $\lambda_L \approx 40$ nm and $L \sim 5$ cm: $N_{\text{eff}} \sim 10^{11}$. A complete derivation of the amplification mechanism from the framework requires additional theoretical development and is left to future work.

8 Connection to the TPST–DGQ Framework

The universal cancellation identity $N(\epsilon) \cdot \epsilon^{d-2} = O(1)$ now operates at nine levels:

Level	**Object**	**Result**
Holography	RT area $\sim \epsilon^{-(d-2)}$	$S_{\text{DG}} = O(1)$
Classical EM	Coulomb energy $\sim \epsilon^{-(d-2)}$	$\langle \mathcal{E} \rangle = O(1)$
Quantum mechanics	Intersection density	$\lvert\psi\rvert^2 = O(1)$
Thermodynamics	Single-sheet entropy	$S_{\text{top}} \geq 0$
EM fields	$\delta F^{(n)} \sim \epsilon^{d-2}$	$\langle F \rangle = F^{\text{std}}$
Gravity	$\Lambda_{\text{bare}} \sim \epsilon^{-2}$	$\Lambda_{\text{obs}} = O(1)$
Statistics	Exchange amplitude	PEP: $\mathcal{A} = 0$
Noncommutativity	$1/\kappa \sim \ell_0/N$	$\kappa = m_P$
Symmetries	Inter-sheet corrections $\sim \langle \Delta_n \rangle$	Poincaré conserved exactly

The gravitoelectromagnetic effect represents the physical consequence of the breaking of one of these symmetries — time-translation invariance — by an external gravitational field. The breaking is controlled by $\langle \Delta_n^{\text{grav}} \rangle \neq 0$, which is non-zero precisely because the gravitational potential correlates the proper-time differences between sheets.

9 Conclusions

We have derived three sets of results from worldline non-injectivity.

Poincaré symmetries. The four conservation laws of classical and quantum physics — energy (Theorem 4.1), momentum (Theorem 4.3), angular momentum (Theorem 4.5), and Lorentz boost invariance (Section 4.4) — emerge from the invariance of the topological average $\mathcal{W} = N^{-1} \sum_n \int \mathcal{L}^{(n)}\, dt$ under the corresponding transformations. The inter-sheet corrections to each conservation law involve $\langle \Delta_n \rangle = 0$ or $\langle \tilde{\Delta}_n \rangle = 0$ and vanish exactly. The conservation laws are derived, not postulated.

Gravitational symmetry breaking. A gravitational field breaks the time-translation invariance of the topological average by producing a non-zero inter-sheet phase gradient:

$$\partial_t \Phi_n^{\text{grav}} = \frac{\gamma^2 v\, GM\, \ell_0}{c^2\, r^2}. \tag{65}$$

This is the geometric mechanism by which gravity enters the electromagnetic sector.

Gravitoelectromagnetic coupling. In the presence of an external magnetic field B_z, the gravitational phase gradient induces an electric field fluctuation (Theorem 6.1):

$$\sigma(\delta E_y) = \frac{GM\, B_z}{c^2\, r^2}. \tag{66}$$

This effect is of first order in G — absent from standard general relativity at this order — and reaches ~ 21 MV/m at the surface of a neutron star with $B \sim 10^{12}$ T.

The central identity remains:

$$\text{Non-injectivity} \iff \text{Finite physics at every level.} \tag{67}$$

Declarations

Conflict of Interest. The author declares no conflicts of interest.

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